MATHEMATICS

Class—XII

Time Allowed : 3 Hours **Maximum Marks : 100**

General Instructions :

(*i*) **All** questions are compulsory.

(*ii*) This question paper contains **29** questions.

(*iii*) Question **1–4** in **Section A** are very short-answer type questions carrying **1** mark each.

(*iv*) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.

(*v*) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.

(*vi*) Question **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each.

Section-A

Question numbers 1 to 4 carry 1 mark each.

1. State the reason why the Relation $R=\{(a, b) : a \leq b^2\}$ on the set **R** of real numbers is not reflexive.

2. If A is a square matrix of order 3 and $|2A| = k|A|$, then find the value of k.

3. If $\vec{a}$ and $\vec{b}$ are two non zero vectors such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$, then find the angle between $\vec{a}$ and $\vec{b}$.

4. If * is a binary operation on the set **R** of real numbers defined by $a * b = a + b - 2$, then find the identity element for the binary operation*.

Section-B

Question numbers 5 to 12 carry 2 marks each.

5. Simplify $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$ for $x < -1$.

6. Prove that the diagonal elements of a skew symmetric matrix are all zeroes.

7. If $y = \tan^{-1}\frac{5x}{1-6x^2}, -\frac{1}{\sqrt{6}}<x<\frac{1}{\sqrt{6}}$, then prove that $\frac{dy}{dx}=\frac{2}{1+4x^2}+\frac{3}{1+9x^2}$.

8. If x changes from 4 to 4·01, then find the approximate change in $\log_e x$.

9. Find $\int\left(\frac{1-x}{1+x^2}\right)^2 e^x\, dx$.

10. Obtain the differential equation of the family of circles passing through the points $(a, 0)$ and $(-a, 0)$.

11. If $|\vec{a}+\vec{b}| = 60$, $|\vec{a}-\vec{b}| = 40$ and $|\vec{a}| = 22$, then find $|\vec{b}|$.

12. If $P(A)=\frac{2}{5}$, $P(B)=\frac{1}{3}$, $P(A \cap B)=\frac{1}{5}$, then find $P(\overline{A}/\overline{B})$.

Section-C

Question numbers 13 to 23 carry 4 marks each.

13. If $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$, then using A^{-1}, solve the following system of equations : $x - 2y = -1$, $2x + y = 2$.

14. Discuss the differentiability of the function

$$f(x) = \begin{cases} 2x-1, & x<\frac{1}{2} \\ 3-6x, & x\geq\frac{1}{2} \end{cases} \text{ at } x = \frac{1}{2}.$$

OR

For what value of k is the following function continuous at $x = -\frac{\pi}{6}$?

$$f(x) = \begin{cases} \dfrac{\sqrt{3}\sin x + \cos x}{x + \dfrac{\pi}{6}}, x \neq -\dfrac{\pi}{6} \\ k, x = -\dfrac{\pi}{6} \end{cases}$$

15. If $x = a \sin pt$, $y = b \cos pt$, then show that $(a^2 - x^2)\, y \dfrac{d^2y}{dx^2} + b^2 = 0.$

16. Find the equation of the normal to the curve $2y = x^2$, which passes through the point (2, 1).

OR

Separate the interval $\left[0, \dfrac{\pi}{2}\right]$ into subintervals in which the function $f(x) = \sin^4 x + \cos^4 x$ is strictly increasing or strictly decreasing.

17. A magazine seller has 500 subscribers and collects annual subscription charges of ₹ 300 per subscriber. She proposes to increase the annual subscription charges and it is believed that for every increase of Re 1, one subscriber will discontinue. What increase will bring maximum income to her ? Make appropriate assumptions in order to apply derivatives to reach the solution. **Write one important role of magazines in our lives.**

18. Find $\displaystyle\int \frac{\sin x}{(\cos^2 x + 1)(\cos^2 x + 4)} dx.$

19. Find the general solution of the differential equation $(1 + \tan y)(dx - dy) + 2xdy = 0.$

OR

Solve the following differential equation :

$$\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0.$$

20. Prove that

$$\vec{a} \cdot \{(\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c})\} = [\vec{a}\,\vec{b}\,\vec{c}].$$

21. Find the value of a so that following lines are skew : $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-a}{4}, \dfrac{x-4}{5} = \dfrac{y-1}{2} = z.$

22. A bag contains 4 green and 6 white balls. Two balls are drawn one by one without replacement. If the second ball drawn is white, what is the probability that the first ball drawn is also white ?

23. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of diamond cards drawn. Also, find the mean and the variance of the distribution.

Section-D

Question numbers 24 to 29 carry 6 marks each.

24. Let $f : [0, \infty) \to R$ be a function defined by $f(x) = 9x^2 + 6x - 5$. Prove that f is not invertible. Modify, only the codomain of f to make f invertible and then find its inverse.

OR

Let * be a binary operation defined on $Q \times Q$ by $(a, b) * (c, d) = (ac, b + ad)$, where Q is the set of rational numbers. Determine, whether * is commutative and associative. Find the identity element for * and the invertible elements of $Q \times Q$.

25. Using properties of determinants, prove that

$$\begin{vmatrix} \dfrac{(a+b)^2}{c} & c & c \\ a & \dfrac{(b+c)^2}{a} & a \\ b & b & \dfrac{(c+a)^2}{b} \end{vmatrix} = 2(a+b+c)^3.$$

OR

If $p \neq 0$, $q \neq 0$ and $\begin{vmatrix} p & q & p\alpha + q \\ q & r & q\alpha + r \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0,$

then, using properties of determinants, prove that at least one of the following statements is true : (*a*) p, q, r are in G.P., (*b*) α is a root of the equation $px^2 + 2qx + r = 0$.

26. Using integration, find the area of the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$.

27. Evaluate the following : $\displaystyle\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx.$

OR

Evaluate $\displaystyle\int_0^4 (x + e^{2x}) dx$ as the limit of a sum.

Find the equation of the plane through the point (4, −3, 2) and perpendicular to the line of intersection of the planes $x - y + 2z - 3 = 0$ and $2x - y - 3z = 0$. Find the point of intersection of the line $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 3\hat{j} - 9\hat{k})$ and the plane obtained above.

29. In a mid-day meal programme, an NGO wants to provide vitamin rich diet to the students of an MCD school. The dietician of the NGO wishes to mix two types of food in such a way that vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food 1 contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food 2 contains 1 unit per Kg of vitamin A and 2 units per kg of vitamin C. It costs ₹ 50 per kg to purchase Food 1 and ₹ 70 per kg to purchase Food 2. Formulate the problem as LPP and solve it graphically for the minimum cost of such a mixture ?